Physics 4A
Chapter 10: Interactions and Potential Energy

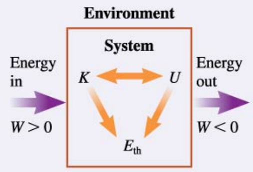
GENERAL PRINCIPLES

The Energy Principle Revisited

- Energy is *transformed* within the system.
- Energy is *transferred* to and from the system by work W.

Two variations of the energy principle are

$$\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}$$
$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$$



Basic energy model

Solving Energy Problems

MODEL Define the system.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart.

SOLVE Use the energy principle:

$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$$

ASSESS Is the result reasonable?

Law of Conservation of Energy

- **Isolated system:** $W_{ext} = 0$. The total system energy $E_{sys} = K + U + E_{th}$ is conserved. $\Delta E_{sys} = 0$.
- **Isolated, nondissipative system:** $W_{ext} = 0$ and $W_{diss} = 0$. The **mechanical energy** $E_{mech} = K + U$ is conserved: $K_i + U_i = K_f + U_f$.

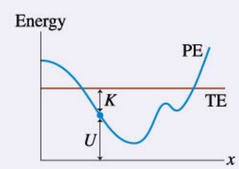
IMPORTANT CONCEPTS

Potential energy, or *interaction energy,* is energy stored inside a system via interaction forces. The energy is stored in *fields.*

- Potential energy is associated only with **conservative forces** for which the work done is independent of the path.
- Work W_{int} by the interaction forces causes $\Delta U = -W_{int}$.
- Force $F_s = -dU/ds = -$(slope of the potential energy curve).
- Potential energy is an energy of the system, not an energy of a specific object.

Energy diagrams show the potential-energy curve PE and the total mechanical energy line TE.

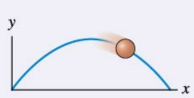
- From the axis to the curve is U. From the curve to the TE line is K.
- **Turning points** occur where the TE line crosses the PE curve.
- Minima and maxima in the PE curve are, respectively, positions of **stable** and **unstable equilibrium.**



APPLICATIONS

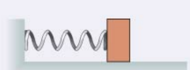
Gravitational potential energy is an energy of the earth + object system:

$$U_G = mgy$$

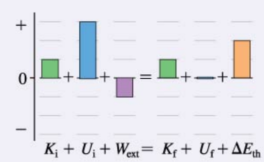


Elastic potential energy is an energy of the spring + attached objects system:

$$U_{Sp} = \tfrac{1}{2}k(\Delta s)^2$$



Energy bar charts show the energy principle in graphical form.



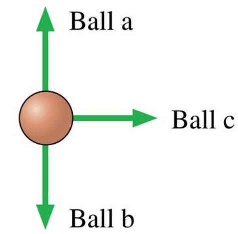
Questions and Example Problems from Chapter 10

Conceptual Question 10.2

Can kinetic energy ever be negative? Can gravitational potential energy ever be negative? For each, give a plausible reason for your answer without making use of any equations.

10.2. No, kinetic energy can never be negative. Kinetic energy is energy of motion. Motion may stop, but it can't be negative. Speed has no direction and cannot be negative. Yes, gravitational potential energy can be negative. Potential energy depends upon position, which can be positive or negative.

Conceptual Question 10.4



The three balls in the figure, which have equal masses, are fired with equal speeds from the same height above the ground. Rank in order, from largest to smallest, their speed v_a, v_b, and v_c as they hit the ground. Explain.

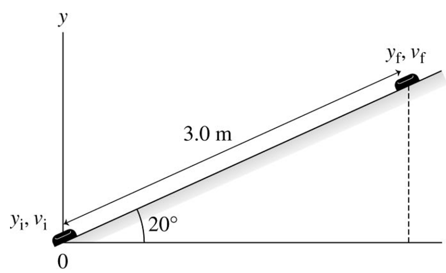
10.4. $v_a = v_b = v_c$. They each start with the same kinetic energy and they each have the same change in potential energy, so they end with the same kinetic energy and, thus, the same speed.

Problem 10.8

What minimum speed does a 100 g puck need to make it to the top of a 3.0-m-long, 20° frictionless ramp?

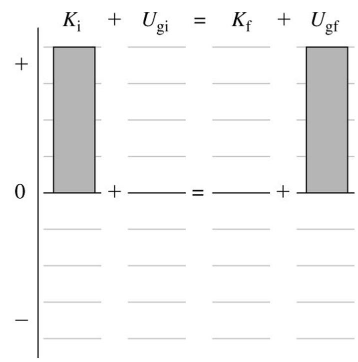
10.8. Model: Model the puck as a particle. Since the ramp is frictionless, the sum of the puck's kinetic and gravitational potential energy does not change during its sliding motion.

Visualize:



Known
$m = 100$ g
$y_i = 0$ m
$y_f = (3.0 \text{ m}) \sin 20° = 1.03$ m
$v_f = 0$ m/s

Find
v_i



Solve: The quantity $K + U_g$ is the same at the top of the ramp as it was at the bottom. The energy conservation equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$\Rightarrow v_i^2 = (0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1.03 \text{ m} - 0 \text{ m}) = 20.2 \text{ m}^2/\text{s}^2 \Rightarrow v_i = 4.5 \text{ m/s}$$

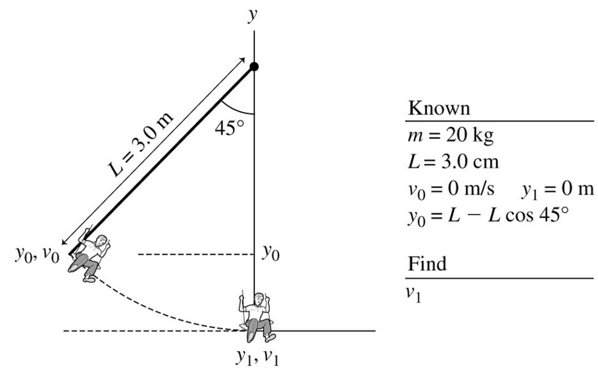
Assess: An initial push with a speed of $4.5 \text{ m/s} \approx 10 \text{ mph}$ to cover a distance of 3.0 m up a $20°$ ramp seems reasonable.

Problem 10.10

A 20 kg child is on a swing that hangs from 3.0-m-long chains. What is her maximum speed if she swings out to a 45° angle?

10.10. Model: Model the child and swing as a particle, and assume the chain to be massless. In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change during the swing's motion.

Visualize:



Solve: The quantity $K + U_g$ is the same at the highest point of the swing as it is at the lowest point. That is, $K_0 + U_{g0} = K_1 + U_{g1}$. It is clear from this equation that maximum kinetic energy occurs where the gravitational potential energy is the least. This is the case at the lowest position of the swing. At this position, the speed of the swing and child will also be maximum. The above equation is

$$\frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow v_1^2 = v_0^2 + 2g(y_0 - y_1)$$
$$\Rightarrow v_1^2 = (0 \text{ m/s})^2 + 2g(y_0 - 0 \text{ m}) \Rightarrow v_1 = \sqrt{2gy_0}$$

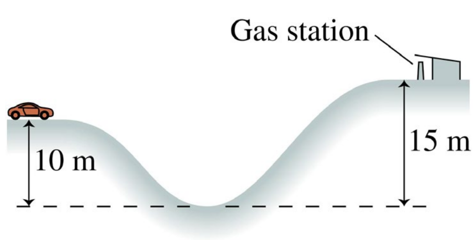
We see from the pictorial representation that

$$y_0 = L - L\cos 45° = (3.0 \text{ m}) - (3.0 \text{ m})\cos 45° = 0.879 \text{ m}$$
$$\Rightarrow v_1 = \sqrt{2gy_0} = \sqrt{2(9.8 \text{ m/s}^2)(0.879 \text{ m})} = 4.2 \text{ m/s}$$

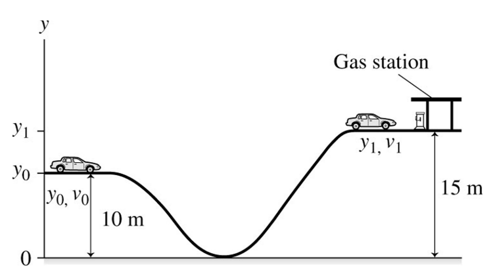
Assess: We did not need to know the swing's or the child's mass. Also, a maximum speed of 4.2 m/s is reasonable.

Problem 10.11

A 1500 kg car traveling at 10 m/s suddenly runs out of gas while approaching the valley shown in the figure. The alert driver puts the car in neutral so that it will roll. What will be the car's speed as it coasts into the gas station on the other side of the valley?



10.11. Model: Model the car as a particle with zero rolling friction and no air resistance. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion.
Visualize:



Solve: The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500\text{ kg})(10.0\text{ m/s})^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 2.22\times10^5\text{ J}$$

The car increases its height to 15 m at the gas station. The conservation of energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ is

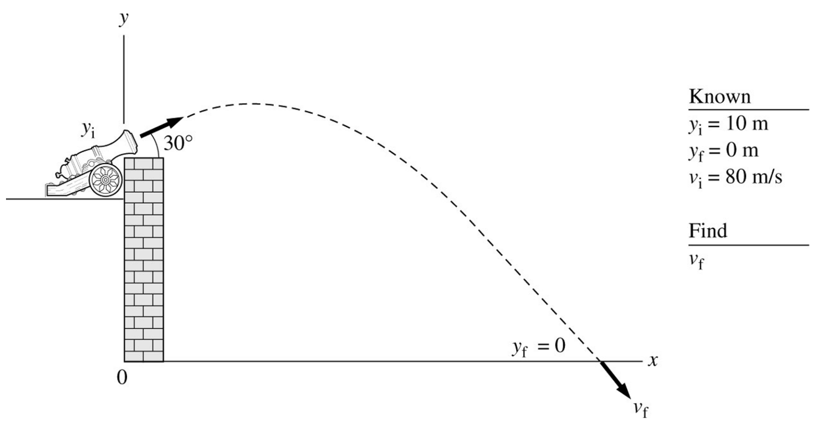
$$2.22\times10^5\text{ J} = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow 2.22\times10^5\text{ J} = \frac{1}{2}(1500\text{ kg})v_1^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})$$
$$\Rightarrow v_1 = 1.4\text{ m/s}$$

Assess: A lower speed at the gas station is reasonable because the car has decreased its kinetic energy and increased its potential energy compared to its starting values.

Problem 10.13

A cannon tilted up at a 30° angle fires a cannon ball at 80 m/s from atop a 10-m-high fortress wall. What is the ball's impact speed on the ground below?

10.13. Model: This is case of free fall, so the sum of the kinetic and gravitational potential energy does not change as the cannon ball falls.
Visualize:



The figure shows a before-and-after pictorial representation. To express the gravitational potential energy, we put the origin of our coordinate system on the ground below the fortress.
Solve: Using $y_f = 0$ and the equation $K_i + U_{gi} = K_f + U_{gf}$ we get

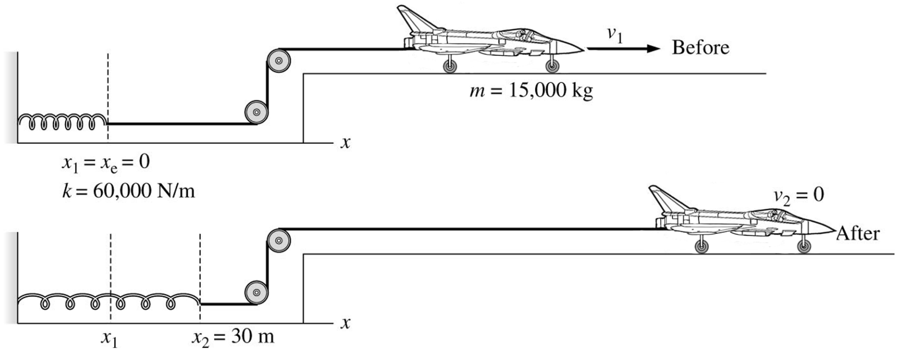
$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 + 2gy_i = v_f^2$$
$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(80\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(10\text{ m})} = 81\text{ m/s}$$

Assess: Note that we did not need to use the tilt angle of the cannon, because kinetic energy is a scalar. Also note that using the energy conservation equation, we can find only the magnitude of the final velocity, not the direction of the velocity vector.

Problem 10.20

As a 15,000 kg jet plane lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

10.20. Model: Model the jet plane as a particle, and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the stretching of the spring, so that mechanical energy is conserved.
Visualize:



The figure shows a before-and-after pictorial representation. The "before" situation occurs just as the jet plane lands on the aircraft carrier and the spring is in its equilibrium position. We put the origin of our coordinate system at the right free end of the spring. This gives $x_1 = x_e = 0$ m. Since the spring stretches 30 m to stop the plane, $x_2 - x_e = 30$ m.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ for the spring-jet plane system is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 30$ m yields

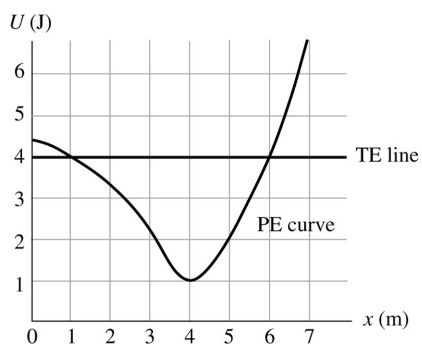
$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_1) = \sqrt{\frac{60{,}000 \text{ N/m}}{15{,}000 \text{ kg}}}(30 \text{ m}) = 60 \text{ m/s}$$

Assess: A landing speed of 60 m/s or ≈ 120 mph is reasonable.

Problem 10.24

The figure below is the potential-energy diagram for a 20-g particle that is released from rest at x = 1.0 m. **(a)** Will the particle move to the right or to the left? How can you tell? **(b)** What is the particle's maximum speed? At what position does it have this speed? **(c)** Where are the turning points of the motion?

10.24. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle is released from rest at $x = 1.0$ m. That is, $K = 0$ at $x = 1.0$ m. Since the total energy is given by $E = K + U$, we can draw a horizontal total energy (TE) line through the point of intersection of the potential energy curve (PE) and the $x = 1.0$ m line. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: (a) We have $E = 4.0$ J and this energy is a constant. For $x < 1.0, U > 4.0$ J and, therefore, K must be negative to keep E the same (note that $K = E - U$ or $K = 4.0 \text{ J} - U$). Since negative kinetic energy is unphysical, the particle cannot move to the left. That is, the particle will move to the right of $x = 1.0$ m.

(b) The expression for the kinetic energy is $E - U$. This means the particle has maximum speed or maximum kinetic energy when U is minimum. This happens at $x = 4.0$ m. Thus,

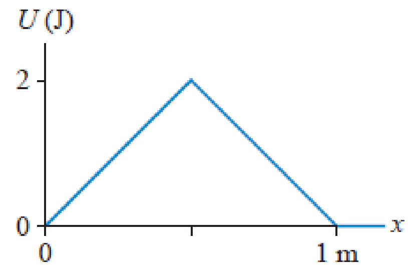
$$K_{max} = E - U_{min} = (4.0\text{ J}) - (1.0\text{ J}) = 3.0\text{ J} \qquad \frac{1}{2}mv_{max}^2 = 3.0\text{ J} \Rightarrow v_{max} = \sqrt{\frac{2(3.0\text{ J})}{m}} = \sqrt{\frac{8.0\text{ J}}{0.020\text{ kg}}} = 17.3\text{ m/s}$$

The particle possesses this speed at $x = 4.0$ m.

(c) The total energy (TE) line intersects the potential energy (PE) curve at $x = 1.0$ m and $x = 6.0$ m. These are the turning points of the motion.

Problem 10.34

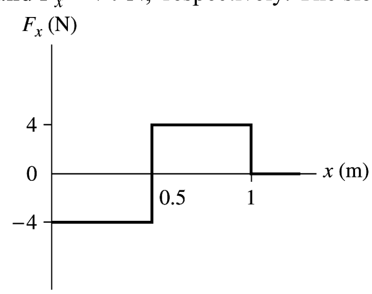
The figure shows the potential energy U of a particle that moves along the x-axis. Draw a graph of the force F_x as a function of position x.



10.34. Model: Use the relationship between a conservative force and potential energy.

Visualize: We will obtain F_x as a function of x by using the calculus technique of differentiation.

Solve: For the interval $0\text{ m} < x < 0.5\text{ m}$, $U = +4x$, and for the interval $0.5\text{ m} < x < 1.0\text{ m}$, $U = -4x + 4$, where x is in meters. The derivatives give $F_x = -4$ N and $F_x = +4$ N, respectively. The slope is zero for $x \geq 1$ m.

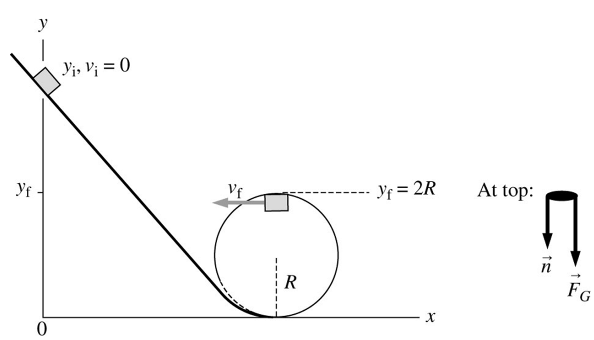


Problem 10.45

A block of mass *m* slides down a frictionless track, then around the inside of a circular loop-the-loop of radius R. From what minimum height *h* must the block start to make it around without falling off? Give your answer as a multiple of R.

10.45. Model: This is a two-part problem. In the first part, we will find the critical velocity for the block to go over the top of the loop without falling off. Since there is no friction, the sum of the kinetic and gravitational potential energy is conserved during the block's motion. We will use this conservation equation in the second part to find the minimum height the block must start from to make it around the loop.

Visualize:



We place the origin of our coordinate system directly below the block's starting position on the frictionless track.

Solve: The free-body diagram on the block implies

$$F_G + n = \frac{mv_c^2}{R}$$

For the block to just stay on track, $n = 0$. Thus the critical velocity v_c is

$$F_G = mg = \frac{mv_c^2}{R} \Rightarrow v_c^2 = gR$$

The block needs kinetic energy $\frac{1}{2}mv_c^2 = \frac{1}{2}mgR$ to go over the top of the loop. We can now use the conservation of mechanical energy equation to find the minimum height *h*.

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

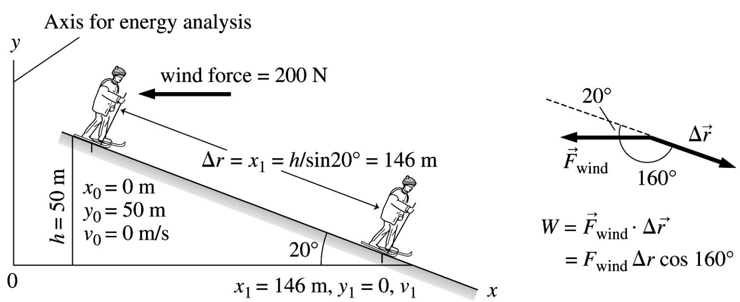
Using $v_f = v_c = \sqrt{gR}$, $y_f = 2R$, $v_i = 0$ m/s, and $y_i = h$, we obtain

$$\frac{1}{2}gR + g(2R) = 0 + gh \Rightarrow h = 2.5R$$

Problem 10.48

Sam, whose mass is 75 kg, straps on his skis and starts down a 50-m-high, 20° frictionless slope. A strong headwind exerts a *horizontal* force of 200 N on him as he skies. Use work and energy to find Sam's speed at the bottom.

10.48. Model: Model Sam strapped with skis as a particle, and apply the law of conservation of energy.
Visualize:



Solve: (a) The conservation of energy equation is

$$K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}$$

The snow is frictionless, so $\Delta E_{th} = 0$ J. However, the wind is an external force doing work on Sam as he moves down the hill. Thus,

$$W_{ext} = W_{wind} = (K_1 + U_{g1}) - (K_0 + U_{g0})$$
$$= \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_0^2 + mgy_0\right) = \left(\tfrac{1}{2}mv_1^2 + 0 \text{ J}\right) - (0 \text{ J} + mgy_0) = \tfrac{1}{2}mv_1^2 - mgy_0$$
$$v_1 = \sqrt{2gy_0 + \frac{2W_{wind}}{m}}$$

We compute the work done by the wind as follows:

$$W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = F_{wind}\Delta r\cos(160°) = (200 \text{ N})(146 \text{ m})\cos(160°) = -27{,}400 \text{ J}$$

where we have used $\Delta r = h/\sin(20°) = 146$ m. Now we can compute

$$v_1 = \sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m}) + \frac{2(-27{,}400 \text{ J})}{75 \text{ kg}}} = 16 \text{ m/s}$$

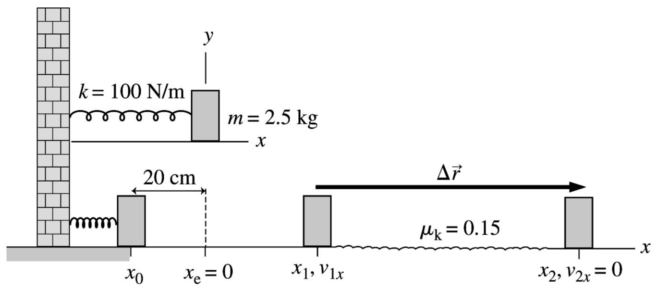
Assess: We used a vertical *y*-axis for energy analysis, rather than a tilted coordinate system, because U_g is determined by its vertical position.

Problem 10.49

A horizontal spring with a spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless, horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction of the box on the surface is 0.15. Use work and energy to find how far the box slides along the rough surface before stopping.

10.49. Model: Assume an ideal spring that obeys Hooke's law. Model the box as a particle and use the model of kinetic friction.

Visualize:



Solve: When the horizontal surface is frictionless, conservation of energy means

$$\tfrac{1}{2}k(x_0 - x_e)^2 = \tfrac{1}{2}mv_{1x}^2 = K_1 \quad \Rightarrow \quad K_1 = \tfrac{1}{2}(100\ \text{N/m})(0.20\ \text{m} - 0\ \text{m})^2 = 2.0\ \text{J}$$

That is, the box is launched with 2.0 J of kinetic energy. It will lose 2.0 J of kinetic energy on the rough surface. The net force on the box is $\vec{F}_{\text{net}} = -\vec{f}_{\text{k}} = -\mu_{\text{k}} mg\hat{i}$. The work–kinetic energy theorem is

$$W_{\text{net}} = \vec{F}_{\text{net}} \cdot \Delta\vec{r} = K_2 - K_1 = 0\ \text{J} - 2.0\ \text{J} = -2.0\ \text{J}$$

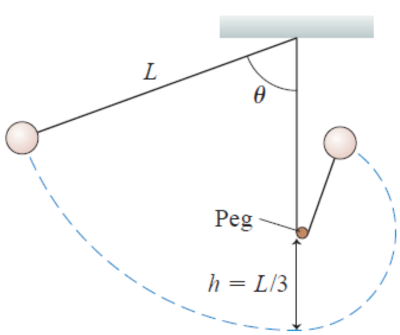
$$(-\mu_{\text{k}} mg)(x_2 - x_1) = -2.0\ \text{J}$$

$$(x_2 - x_1) = \frac{2.0\ \text{J}}{\mu_{\text{k}} mg} = \frac{2.0\ \text{J}}{(0.15)(2.5\ \text{kg})(9.8\ \text{m/s}^2)} = 0.54\ \text{m}$$

Assess: Because the force of friction transforms kinetic energy into thermal energy, energy is transferred out of the box into the environment. In response, the box slows down and comes to rest.

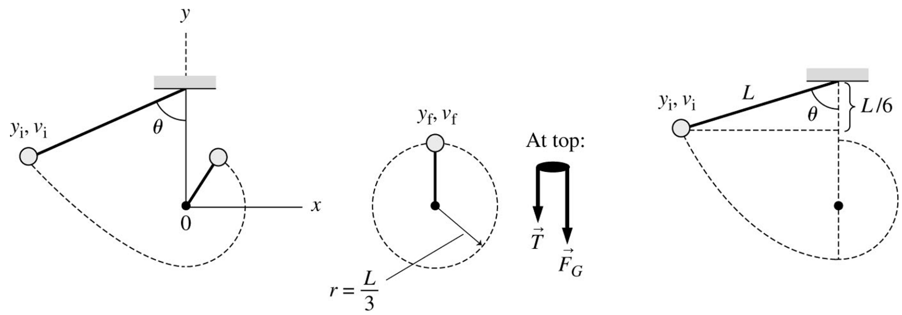
Problem 10.69 (challenging)

A pendulum is formed from a small ball of mass m on a string of length L. As the figure shows, a peg is height h = L/3 above the pendulum's lowest point. From what minimum angle θ must the pendulum be released in order for the ball to go over the top of the peg without the string going slack?



10.69. Model: This is a two-part problem. In the first part, we will find the critical velocity for the ball to go over the top of the peg without the string going slack. Using the energy conservation equation, we will then obtain the gravitational potential energy that gets transformed into the critical kinetic energy of the ball, thus determining the angle θ.

Visualize:



We place the origin of our coordinate system on the peg. This choice will provide a reference to measure gravitational potential energy. For θ to be minimum, the ball will just go over the top of the peg.

Solve: The two forces in the free-body force diagram provide the centripetal acceleration at the top of the circle. Newton's second law at this point is

$$F_G + T = \frac{mv^2}{r}$$

where T is the tension in the string. The critical speed v_c at which the string goes slack is found when $T \to 0$. In this case,

$$mg = \frac{mv_C^2}{r} \Rightarrow v_C^2 = gr = gL/3$$

The ball should have kinetic energy at least equal to

$$\frac{1}{2}mv_C^2 = \frac{1}{2}mg\left(\frac{L}{3}\right)$$

for the ball to go over the top of the peg. We will now use the conservation of mechanical energy equation to get the minimum angle θ. The equation for the conservation of energy is

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c, y_f = \frac{1}{3}L, v_i = 0,$ and the above value for v_C^2, we get

$$\frac{1}{2}mg\frac{L}{3} + mg\frac{L}{3} = mgy_i \Rightarrow y_i = \frac{L}{2}$$

That is, the ball is a vertical distance $\frac{1}{2}L$ above the peg's location or a distance of

$$\left(\frac{2L}{3} - \frac{L}{2}\right) = \frac{L}{6}$$

below the point of suspension of the pendulum, as shown in the figure on the right. Thus,

$$\cos\theta = \frac{L/6}{L} = \frac{1}{6} \Rightarrow \theta = 80.4°$$